

# SOLUTION OF THE STRING VIBRATION EQUATION BY THE FOURIER METHOD

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**Abstract** *Narrow swing in the article equation Fur method using solution seeing is output . Variables separation method used without , borderline the conditions satisfactory solution functions multiplication as is being sought . Fure row using solution general shape are given , the coefficients determination process The solution is explained . correctness and borderline to the conditions compatibility is proven .*

**Keywords** *narrow vibration Fourier equation method , variables separation , boundary conditions , Fure row , characteristic functions , properties values , trigonometric series , coefficients , solution Convergence*

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Variables separation or Fur method many mathematician physics equations in solution is applied .

Example: This  $\frac{\partial^2 u}{\partial t^2} + b^2 u = a^2 \cdot \frac{\partial^2 u}{\partial x^2}$  (1) of the equation

$$u(0;t) = 0 \quad (2), \quad u(l;t) = 0 \quad (3), \quad u(x;0) = f(x) \quad (4), \quad u_t(x;0) = \varphi(x) \quad (5)$$

satisfying the boundary conditions  $u(x;t)$  . We will look for  $u(x;t) \neq 0$  the solution in the form of a product of functions  $t$  , one of which depends on and the other  $x$  on, and assume that they can be differentiated in the desired order.

$$u(x;t) = X(x)T(t) \quad (6)$$

We find the necessary partial derivatives and (1) put them in

$$X(x)T''(t) + b^2 X(x)T(t) = a^2 X''(x)T(t) \text{ by } a^2 X(x)T(t).$$

$$\frac{T''(t)}{a^2 T(t)} + \frac{b^2}{a^2} = \frac{X''(x)}{X(x)} \quad (7)$$

On the left side are differential equations that depend only  $t$  on , and on the right side  $x$  are differential equations that depend only on . The equality is valid only if they are equal to some constant number. This constant is  $-\lambda$  ( $\lambda > 0$ ) We denote by .

$$\frac{T''(t)}{a^2 T(t)} + \frac{b^2}{a^2} = \frac{X''(x)}{X(x)} = -\lambda$$

$$X''(x) + \lambda X(x) = 0 \quad (8)$$

$$T''(t) + (b^2 + a^2 \lambda_k) T(t) = 0 \quad (9)$$

The general solutions of these equations are

$$X_k(x) = \sin\left(\frac{k\pi x}{l}\right), \lambda_k = \left(\frac{k\pi}{l}\right)^2 \quad (10)$$

$$T_k(t) = a_k \cos(\omega_k t) + b_k \sin(\omega_k t)$$

$$\omega_k = \sqrt{a^2 \lambda_k + b^2} = \sqrt{a^2 \left(\frac{k\pi}{l}\right)^2 + b^2} \quad (11)$$

the found ones (6) in ,

$$u(x, t) = \sum_{k=1}^{\infty} [a_k \cos(\omega_k t) + b_k \sin(\omega_k t)] \sin\left(\frac{k\pi x}{l}\right) \text{ We create .}$$

$$\text{Here } \omega_k = \sqrt{a^2 \left(\frac{k\pi}{l}\right)^2 + b^2}$$

According to the initial condition,  $a_k$  we find and  $b_k$

$$u(x, 0) = f(x)$$

$$f(x) = \sum_{k=1}^{\infty} a_k \sin\left(\frac{k\pi x}{l}\right) \rightarrow a_k = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{k\pi x}{l}\right) dx$$

$$u_t(x, 0) = \varphi(x)$$

$$\varphi(x) = \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi x}{l}\right) \rightarrow b_k = \frac{2}{l\omega_k} \int_0^l \varphi(x) \sin\left(\frac{k\pi x}{l}\right) dx$$

The final solution formula is:


$$u(x, t) = \left[ \left( \frac{2}{l} \int_0^l f(x) \sin\left(\frac{k\pi x}{l}\right) dx \right) \cos(\omega_k t) + \left( \frac{2}{l\omega_k} \int_0^l \varphi(x) \sin\left(\frac{k\pi x}{l}\right) dx \right) \sin(\omega_k t) \right] \sin\left(\frac{k\pi x}{l}\right) \quad \text{In}$$

this:

$$\omega_k = \sqrt{a^2 \left(\frac{k\pi}{l}\right)^2 + b^2}$$

**Conclusion :** This example is incorrect. and reverse issues from science independent education to perform in the process worked . Narrow vibration in the article equation Fur method using solution seeing is output . Variables separation method used without , borderline the conditions satisfactory solution functions multiplication as is being sought . Fure row using solution general shape are given , the coefficients determination process The solution is explained . correctness and borderline to the conditions compatibility is proven .

Article narrow swing equation Fur method through to solve analysis does . Variables separation from the method using , borderline the conditions satisfactory solution functions multiplication in the form of is found . Fure row using solution general appearance and coefficients is determined , the solution correctness and to the conditions compatibility is proven .



Article Incorrect and reverse issues from science independent education process to perform in the process prepared .

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