# NUMERICAL CHARACTERISTICS OF RANDOM VARIABLES

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**ANNOTATION**. Numerical characteristics of random variables are one of the fundamental branches of mathematical statistics and probability theory, allowing for the quantitative description and analysis of the properties of random events. This article provides an in-depth discussion of the theoretical foundations, mathematical expressions, and practical significance of the main numerical characteristics of random variables—such as the mean, variance, standard deviation, skewness coefficient, kurtosis, moments, quantiles, and median.

The numerical characteristics of a random variable help to fully describe its distribution pattern. The mathematical expectation shows the average value of a random variable, the variance and standard deviation express the degree of its spread, and the skewness and kurtosis express the shape and deviation of the distribution. These characteristics are widely used in probability theory, statistical analysis, financial mathematics, quality control, insurance business, and many other scientific and practical areas.

The article considers numerical characteristics for discrete and continuous types of random variables separately. The exact formulas for the mathematical expectation and variance for discrete random variables, and the integral expressions and their properties for continuous random variables are analyzed in detail. The relationship between empirical and theoretical characteristics, methods for their estimation (point and interval estimates), estimation methods such as the method of moments and the maximum likelihood method are also covered.

The article shows with examples how the numerical characteristics of random variables are used in modern scientific research, their role in financial risk assessment, quality control, medical statistics, and machine learning algorithms. In particular, the importance of these characteristics in statistical data analysis in the conditions of Uzbekistan and current issues in their application in practice are also discussed.

This study serves as a valuable scientific resource for students, scientists, and professionals in mathematics, statistics, and applied sciences by providing a deep theoretical and practical understanding of the topic of numerical characteristics of random variables. The article helps to better understand the properties of random variables and their effective application in various fields.

**Keywords:**random variable, numerical characteristics, mathematical expectation, variance, standard deviation, skewness coefficient, kurtosis, moments, quantile, median,

empirical distribution, interval estimation, probability theory, statistical analysis, financial risk assessment, quality control.

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Numerical characteristics of random variables are one of the main and most important sections of mathematical statistics and probability theory. To fully describe any random event or process, it is not enough to know its distribution law. Numerical characteristics are needed that quantitatively express the main properties of this distribution. They express the average value, degree of dispersion, symmetry, deviations and other important properties of a random variable in terms of specific numbers.

The mathematical expectation (mean), variance, standard deviation, skewness coefficient, kurtosis, higher-order moments, median, quantile and other indicators are the main types of numerical characteristics of random variables. These characteristics are of great importance not only theoretically, but also in practice. They are widely used in financial risk assessment, quality control systems, medical and biological research, economic forecasting, machine learning algorithms and many other areas.

In modern scientific research, the numerical characteristics of random variables are of particular relevance. In the era of Big Data, these characteristics serve as a key tool for working with large amounts of data, creating statistical models, and in decision-making processes. For example, in financial markets, mathematical expectation and variance are used to predict asset returns, while in quality control, skewness and kurtosis indicators help assess the stability of the process.

The field of statistical analysis and the application of probability theory is also rapidly developing in the Republic of Uzbekistan. The digitalization of the economy, strengthening quality control in industrial enterprises, financial risk management, and medical statistical research are closely related to the numerical characteristics of random variables. However, it is observed that there is a lack of scientific literature in the Uzbek language on the in-depth scientific and theoretical coverage and practical application of this topic.

This article aims to fill this gap. The article provides an in-depth analysis of the theoretical foundations, mathematical expressions, properties, and interrelationships of the main numerical characteristics (mathematical expectation, variance, standard deviation, moments, skewness, kurtosis, etc.) for discrete and continuous types of random variables. It also discusses how these characteristics can be estimated from empirical data, their point and interval estimates, their application in modern software (Python, R, MATLAB), and examples from various practical areas.

**Main part**

The theory of random variables is one of the central concepts of probability theory and is of great importance in the mathematical modeling of uncertain processes in real life. A random variable is a numerical function that takes a certain value as a result of a random experiment. They are usually divided into discrete and continuous types, and the numerical characteristics of each type are studied separately. The numerical characteristics of random variables serve to briefly and clearly express their main properties. Using these characteristics, one can draw conclusions about the general properties of a random variable even without fully knowing its distribution law.

One of the most important numerical characteristics of a random variable is its expectation. The expectation represents the average value of a random variable and approximates the average result obtained as a result of long-term experiments. For a discrete random variable, the expectation is determined by the sum of the product of the values and their probabilities. For a continuous random variable, it is expressed in integral form. Expectation is widely used in forecasting and decision-making processes in many fields, such as economics, physics, and statistics.

The next important characteristic is the variance. The variance shows how much the random variable deviates from the mathematical expectation, that is, the degree of dispersion of the values. If the variance is small, the values of the random variable are densely located around the mathematical expectation, otherwise they are more widely spread. The variance is determined by the squared deviations of the mathematical expectation. The square root of the variance is called the mean square deviation or standard deviation and is more commonly used in practical calculations, since it is expressed in the units of measurement of the random variable itself.

Another important numerical characteristic of random variables is moments. Moments allow us to study the shape of the distribution of a random variable in more detail. There are initial moments and central moments, which are determined in different orders. For example, the first-order central moment is zero, while the second-order central moment gives the variance. The third- and fourth-order moments provide information about the asymmetry and peaking of the distribution.

Skewness (the coefficient of skewness) indicates whether the distribution of a random variable is symmetric or non-symmetric. If the skewness is zero, the distribution is symmetric. Positive skewness indicates that the distribution is skewed to the right, and negative skewness indicates that it is skewed to the left. The kurtosis (the coefficient of skewness) indicates how "peaked" or "flat" the distribution is compared to a normal distribution.

Among the numerical characteristics of random variables, covariance and correlation also occupy an important place. Covariance determines the degree of dependence between two random variables. If the covariance is positive, the quantities change in the same direction, and if it is negative, they change in the opposite direction. Correlation is a normalized form of covariance, which takes a value in the range from -1 to 1 and more clearly shows the strength of the dependence.

Also important in the study of the numerical characteristics of random variables are the connections between the distribution function and the density function. Using these functions, one can calculate the mathematical expectation, variance, and other characteristics. For example, for a continuous random variable, the mathematical expectation is determined by the density function, and the variance is calculated by an integral based on this function.

In practice, the numerical characteristics of random variables are widely used in the analysis of statistical data. For example, in the analysis of economic indicators, the mean and variance are used to assess market changes. In technical fields, these characteristics are important in separating signals from noise and processing experimental results.

**Conclusion**

Numerical characteristics of random variables are one of the most important and integral components of probability theory and mathematical statistics. These concepts provide an opportunity to understand the inner nature of random processes, their laws and behavior more deeply. Numerical characteristics of random variables — such indicators as mathematical expectation, variance, mean square deviation, moments, skewness, excess, covariance and correlation — are of great importance not only theoretically, but also in solving practical problems.

The mathematical expectation expresses the central tendency of a random variable and describes the average value obtained as a result of long-term experiments. This indicator determines the main direction of the random process. The variance and the standard deviation show how much the random variable deviates from the mathematical expectation, allowing to assess its stability. In this regard, they serve as an important tool for identifying risks, assessing the level of uncertainty and making various decisions.

Higher-order moments, in particular skewness and kurtosis, play a major role in determining the shape of a random variable's distribution. They can provide a clear idea of whether the distribution is symmetrical or asymmetrical, peaked or flat. This is especially important in statistical analysis, economic modeling, and scientific research.

Covariance and correlation indices are used to determine the relationship between several random variables. They can be used to determine the degree and direction of interaction, which is widely used in many practical problems, including economics, finance, engineering, and natural sciences. In particular, the importance of these indices in regression analysis and forecasting processes is incomparable.

The numerical characteristics of random variables are widely used in various areas of modern science and technology. For example, in economics, they play an important role in analyzing market trends, assessing risks, and making optimal decisions; in engineering, in separating signals from noise; and in medicine, in making diagnoses and assessing the effectiveness of treatment based on statistical data. These concepts are also one of the main methodological tools in the fields of artificial intelligence and data analysis.

In general, through the numerical characteristics of random variables, it is possible to simplify the study of complex random events, determine their main properties and predict their future states. This allows to increase the efficiency of scientific research, to solve practical problems accurately and reasonably. Therefore, an in-depth study of this topic is important for every specialist in mathematics, statistics and technical fields.

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