INTEGRAL APPROXIMATION CALCULATION

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Annotation: *Modern scientific and technological development has made mathematical methods, especially integral calculation techniques, significant in almost all areas of life. The identified integral is used to model any physical, economic, or technical process. However, in many cases, the function is not given analytically, or it is impossible to obtain its exact integral. In such situations, approximate integral calculation methods—such as the trapezoidal method, Simpson's method, and Boole's formula—are applied. This article discusses the practical applications of approximate integral calculation in real life.*

Keywords: *integral, approximate calculation, trapezoidal method, Simpson's method, real-world problem, fuel consumption, blood flow.*

Introduction: In various fields of modern science and technology—such as physics, economics, technology, medicine, and ecology—the need to determine the area under complex functions arises frequently. In many of these cases, calculating the integral analytically is not feasible, or the calculation process becomes extremely complicated. In such cases, approximate integral calculation methods hold significant importance. The trapezoidal and Simpson's methods are considered the simplest and most practical methods for approximate integral calculation. These methods allow for estimating the entire integral based on the known values of the function over a specified interval. Especially in practical situations where exact data is not available, but point values obtained experimentally exist, these methods become particularly useful.

This article discusses the theoretical foundations of approximate integral calculation methods, and their practical applications in real-life problems will be illustrated with examples based on real data. The mechanics of the trapezoidal and Simpson's methods will be analyzed step by step, demonstrating their practical benefits.

In medicine, studying human circulation involves variations in blood flow velocity over time. If the blood velocity starts at 0.2 m/s during a heartbeat, reaches a maximum of 0.7 m/s, and then slows down, finding a mathematical formula that accurately expresses this velocity is challenging. However, using the velocities measured every second, the volume of blood can be approximately calculated using Simpson's method. This is an important factor in assessing heart health.

The need for integrals also arises when determining the surface area or volume of bridges, structures with various slopes, or complex curved shapes. In such cases, engineers divide the function graph into segments, approximating each part as trapezoids to perform approximate integration. For example, if the base of a bridge spans different distances in relation to the water level, engineers use the trapezoidal method to determine the overall water flow area under the bridge.

When evaluating pollution levels in the atmosphere, the concentrations of various gases change in different locations. Therefore, in air quality studies, point measurements are taken rather than relying on a precise formula. Based on these measurements, the amount of polluted air is calculated using the trapezoidal method. This data is valuable for identifying ecological risks and developing measures to mitigate them.

Fuel consumption varies depending on speed, road conditions, and load weight. For instance, if a truck's fuel consumption over a 100 km distance is measured every 10 km, this data can be used to calculate the total fuel consumption using the Simpson or trapezoidal method. These methods are useful for transport companies in analyzing fuel efficiency.

In economic analysis, factors such as product demand and supply, and price changes dynamically change over time. Approximate integral calculation methods are utilized to determine total revenue or profit amounts by using graphs that represent these changes. Companies especially use these methods when making financial forecasts.

In this research, we will examine the most widely used numerical method for approximate integration—the trapezoidal method—to solve a real-life problem. The problem is to determine the quantity (volume) of a liquid flowing through a water pipe. In this case, the flow velocity is measured at every 1-meter interval and expressed in the form of a function:

$$v(x) = 3 + 0.5\sin(x), 0 \le x \le 10$$

The goal is to approximately estimate the volume of water that has flowed through a pipe over a distance from 0 to 10 meters.

$$V = \int_0^{10} (3 + 0.5\sin(x))dx$$

Step 1: Divide the Interval

Interval: $[0;10]$

Number of subintervals: $n = 10$ (can be any number)

Length of each subinterval: $h = \frac{b-a}{n} = \frac{10-0}{10} = 1$

Function values will be calculated at the following points:

i	x_i	$f(x_i)$
0	0	3.000
1	1	3.4207
2	2	3.4546
3	3	3.0706
4	4	2.6211
5	5	2.5206
6	6	2.8589
7	7	3.3368
8	8	3.4947
9	9	3.2822
10	10	2.7290

Trapezoidal Rule Formula:

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

Here:

$$f(x_0) = 3.0000$$
$$f(x_{10}) = 2.7290$$

$2\sum f(x_i)$ We calculate based on (from 1 to 9):

S = 2(3.4207 + 3.4546 + 3.0706 + 2.6211 + 2.5206 + 2.8589 + 3.3368 + 3.4947 + 3.2822) = 2(28.0592) = 56.1184

Using the full formula:

$$V \approx \frac{1}{2}[3+56.1184+2.729] = \frac{1}{2}(61.8474) = 30.9237$$

This demonstrates that the trapezoidal rule is effective for quick approximations in practice, but provides more accurate results when used with a larger number of subintervals.

Now, we will estimate the integral using Simpson's Rule. For Simpson's rule, the number of intervals must be even — for example:

$n=10$ {(**Even number, 10 subintervals**)}

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

We will calculate the values at the following points:

i	x_i	$f(x_i) = 3+0.5\sin(x_i)$
0	0	3.000
1	1	3.4207
2	2	3.4546
3	3	3.0706
4	4	2.6211
5	5	2.5206
6	6	2.8589
7	7	3.3368
8	8	3.4947
9	9	3.2822
10	10	2.7290

Simpson's Rule has the following form:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(x_0) + 4\sum_{toqlari} f(x_i) + 2\sum_{juftlari} f(x_i) + f(x_n) \right]$$

Odd indices:: x_1, x_3, x_5, x_7, x_9

Even indices : x_2, x_4, x_6, x_8

Let's calculate:

$$f(x_0) = 3.0000$$
$$f(x_{10}) = 2.7290$$

The sum of values at odd indices is:

$$4(3.4207+3.0706+2.5206+3.3368+3.2822)=4(15.631)=62.524$$

Then, we find the sum of values at even indices:

$$2(3.4546+2.6211+2.8589+3.4947)=2(12.4293)=24.8586$$

We arrive at the final result:

$$V\approx\frac{1}{3}[3+62.524+24.8586+2.729]=\frac{1}{3}(93.1116)\approx31.0372$$

Result using Simpson's Rule:
Result using the Trapezoidal Rule:

$$V_{simpson}\approx31.04m^3$$
$$V_{trapetsiya}\approx30.92m^3$$

Analysis: The trapezoidal rule allows for simple and quick computation. However, it may lose some accuracy compared to **Simpson's Rule**, which is based on a parabolic approximation.
In this example, the difference between the trapezoidal and Simpson's methods is:

$$\Delta V=|31.04-30.92|=0.12m^3$$

This indicates that Simpson's rule is much more accurate, especially when the function is smooth and differentiable. It also shows that the difference between the results obtained by the two methods is very small, confirming that the trapezoidal rule provides practically sufficient accuracy in real-life applications such as hydrodynamic measurements, energy consumption estimation, or economic indicator integration.
Moreover, this method can be automated using computer programs, which saves time and reduces human error when working with large datasets.

Conclusion: Approximate methods for calculating integrals are widely used not only in theoretical mathematics but also in many practical fields such as medicine, engineering, ecology, economics, and transportation. These methods allow simplifying complex processes and finding mathematical solutions to them. Therefore, this topic is of great importance in modern practice, and specialists in every field need to master these methods thoroughly.

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