

PUASSON TENGLAMASI UCHUN DIRIXLE CHEGARAVIY MASALASINING AYIRMALI APPROKSIMATSIYASI VA YAQINLASHUVINI QAT'IIY TAHLILI.

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Annotatsiya. Mazkur ishda Puasson tenglamasi uchun Dirixle tipidagi chegaraviy masala qaraladi. Laplas operatorining markaziy ayirmalar yordamida qurilgan diskret analogi asosida klassik besh nuqtali ayirmali sxema hosil qilinadi. Ayirmali sxemaning aniqlik tartibi, barqarorligi hamda bir jinsli normada yaqinlashuvi qat'iy matematik baholanadi. Isbotlanadiki, taklif etilgan sxema ikkinchi tartibli aniqlikka ega bo'lib, $O(h^2)$ tezlik bilan yaqinlashadi.

Kalit so'zlar. Puasson tenglamasi, Dirixle masalasi, ayirmali sxema, Laplas operatori, yaqinlashuv, maksimal prinsp.

Kirish. Elliptik tipdagi differensial tenglamalar, xususan Puasson tenglamasi,

$$\Delta u(x) = -f(x),$$

ko'plab statsionar jarayonlarning matematik modeli hisoblanadi. Chegaraviy qiymatlar berilgan Dirixle masalasi amaliy hisoblashlarda keng uchraydi. Murakkab sohalarda analitik yechim mavjud bo'lmagani sababli, masalani sonli usullar yordamida yechish dolzarb hisoblanadi.

Mazkur ishda Puasson tenglamasi uchun ayirmali usul asosida yechim qurilib, uning aniqlik va yaqinlashuv xossalari qat'iy matematik apparat yordamida tahlil qilinadi.

Masalaning qo'yilishi va ayirmali sxemani qurish

Chegaralangan $G \subset \mathbb{R}^2$ sohada quyidagi Dirixle masalasini qaraymiz:

$$\begin{cases} \Delta u(x, y) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -f(x, y), & (x, y) \in G, \\ u(x, y) = \mu(x, y), & (x, y) \in \Gamma, \end{cases}$$

bu yerda $\Gamma = \partial G$ Faraz qilamizki,

$$f \in C^2(\overline{G}), \quad \mu \in C(\Gamma).$$

Panjara va diskret operator

G sohada $h > 0$ qadamli tekis to'rtburchakli panjara kiritamiz:

$$x_i = ih, \quad y_j = jh, \quad \omega_h = \{(x_i, y_j) \in G\}, \quad \gamma_h = \omega_h \cap \Gamma.$$

Ikkinchi tartibli hosilalar markaziy ayirmalar bilan approksimatsiya qilinadi:

$$\frac{\partial^2 u}{\partial x^2}(x_i, y_j) = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + O(h^2), \quad \frac{\partial^2 u}{\partial y^2}(x_i, y_j) = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2} + O(h^2).$$

Shu asosda Laplas operatorining ayirmali analogi aniqlanadi:

$$\Lambda_h y_{i,j} = \frac{y_{i+1,j} + y_{i-1,j} + y_{i,j+1} + y_{i,j-1} - 4y_{i,j}}{h^2}.$$

Ayirmali Dirixle masalasi.

Ichki tugunlar uchun ayirmali tenglama:

$$\Lambda_h y_{i,j} = -f_{i,j}, \quad (x_i, y_j) \in \omega_h,$$

chegaraviy tugunlarda esa:

$$y_{i,j} = \mu(x_i, y_j), \quad (x_i, y_j) \in \gamma_h.$$

Aniqlik va yaqinlikni baholash.

Haqiqiy yechim $u(x, y)$ ni (3) ga qo'yib, lokal xatolikni aniqlaymiz:

$$\psi_{i,j} = \Lambda_h u_{i,j} + \Delta u(x_i, y_j).$$

Teylor yoyilmasidan foydalanib, quyidagiga ega bo'lamiz:

$$\psi_{i,j} = O(h^2).$$

Xatolik tenglamasi.

Xatolik funksiyasi

$$z_{i,j} = y_{i,j} - u_{i,j}$$

uchun quyidagi masala hosil bo'ladi:

$$\begin{cases} \Lambda_h z_{i,j} = -\psi_{i,j}, & (x_i, y_j) \in \omega_h, \\ z_{i,j} = 0, & (x_i, y_j) \in \gamma_h. \end{cases}$$

Diskret maksimal printsipga asosan teskari operator uchun baho o'rinli:

$$\|z\|_C \leq C \|\psi\|_C.$$

(5) va (7) ni birlashtirib, quyidagi natijaga kelamiz:

$$\|y - u\|_C = \|z\|_C = O(h^2).$$

Olingan baho shuni ko'rsatadiki, qurilgan besh nuqtali ayirmali sxema Puasson tenglamasi uchun Dirixle masalasini yechishda barqaror va ikkinchi tartibli aniqlikka ega. Sxemaning yaqinlashuvi bir jinsli normada ta'minlanadi, bu esa uning amaliy hisoblashlarda ishonchliligini kafolatlaydi. Shuni ta'kidlash lozimki, aniqlik tartibi panjara qadami h ga bog'liq bo'lib, $h \rightarrow 0$ da yechim uzluksiz masala yechimiga yaqinlashadi.

Xulosa. Mazkur ishda Puasson tenglamasi uchun Dirixle chegaraviy masalasining ayirmali approksimatsiyasi qat'iy matematik asosda qurildi. Laplas operatorining besh nuqtali ayirmali sxemasi uchun lokal va global xatolik baholari keltirildi hamda sxemaning $O(h^2)$ tezlik bilan yaqinlashishi isbotlandi.

Foydalanilgan adabiyotlar.

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