AN INTERVAL EXTENSION OF THE POTENTIAL METHOD FOR TRANSPORTATION PROBLEMS

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Annotation. *This article presents the process of obtaining numerical results for defining the base and initial plans. An algorithm for determining the optimal interval plan is proposed. The potential-based approach is adapted to an interval version of the method.*

Keywords: *interval plan, base plan, optimization algorithm, potential method, numerical analysis.*

One of the essential factors for sustainable economic growth is the use of quantitative analysis based on mathematical methods and modern computer technologies, which enable sound economic decision-making. Mathematical programming provides the theoretical foundation for such analyses and includes linear, nonlinear, and dynamic programming.

The transportation problem, first introduced by A. N. Tolstoy (1930) and further developed by L. V. Kantorovich (1939), represents one of the key models in optimization theory. Significant progress was achieved with the introduction of the simplex method by G. B. Dantzig (1949) and the potential method by L. V. Kantorovich and M. K. Gavurin (1949). Later, H. W. Kuhn and A. W. Tucker (1951) established optimality conditions for nonlinear problems, expanding the field of mathematical programming.

The transportation problem serves as a universal model for various practical applications. When bilateral constraints are replaced with interval analysis, the problem transforms into finding an interval matrix that contains all possible optimal solutions within given parameter ranges. This approach provides a flexible framework for solving real-world optimization problems under uncertainty.

Let the cost of transporting the same cargo unit from supplier $\mathbf{d}_{ij}$ (item A_i) to consumer j (item C_j) be $\mathbf{x}_{ij} = [\underline{x}_{ij}, \overline{x}_{ij}]$; i is the amount of cargo delivered from the supplier to the second consumer; a_i is the amount of cargo at the point; A_i is the requirement for the product in the paragraph.

Therefore, the following interval problem arises

$$|\mathbf{F}| = \left| \sum_{i=1}^{m} \sum_{j=1}^{n} \mathbf{d}_{ij} x_{ij} \right| \rightarrow \min, \qquad (1)$$

$$\sum_{j=1}^{n} \bar{x}_{ij} = a_i \quad (i = \overline{1, m}) \quad , \tag{2}$$

$$\sum_{i=1}^{m} \mathbf{x}_{ij} = \mathbf{b}_j \quad (j = \overline{1, n}) \quad , \tag{3}$$

$$\mathbf{x}_{ij} \subseteq \mathbf{w}_{ij} \quad (i = \overline{1, m}, \quad j = \overline{1, n}) \quad , \tag{4}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} \bar{b}_j \quad , \tag{5}$$

where $\mathbf{X} = (x_{ij}) = ([\underline{x}_{ij}, \ \bar{x}_{ij}])$ is the interval matrix of transport, $\mathbf{W} = (w_{ij}) = ([\underline{w}_{ij}, \ \bar{w}_{ij}])$ is the interval matrix of transport constraints

$$\sum_{i=1}^{m} \underline{w}_{ij} \le \underline{b}_j \le \bar{b}_j \le \sum_{i=1}^{m} \bar{w}_{ij}, \qquad (j = \overline{1, n}), \tag{6}$$

Definition. The support plan for problems 1 (1) - (6) is a $\mathbf{X}$ matrix that satisfies conditions (2) - (6).

It should be noted that the problem (1) - (5), under the condition (6), can be interpreted as a model for a situation where we know the claims and believe that we should satisfy them to the greatest extent possible.

In general, instead of (6), from a mathematical point of view, any choice from the relations

$$\underline{b}_j \ne \underline{w}_{ij}, \ \ \bar{w}_{ij} \ne \bar{b}_j \quad (j = \overline{1, \ n}) . \tag{7}$$

However, the cases (7) are analyzed below, based on traditional criteria for solving the transport problem, including consideration of the uncertainty interval of the parameters:

Theorem 1. Any degenerate problem corresponding to the interval problem (1) - (6) can be solved.

Evidence

$$\sum_{j=1}^{n} \underline{b}_j = \underline{s} \quad , \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} \bar{b}_j = \bar{s} \quad .$$

Then, according to the condition of theorem $\underline{x}_{ij} = \bar{x}_{ij} = x_{ij}$, and for the quantities $x_{ij} \in [(a_i \underline{b}_j) / \underline{s}, \ (a_i \bar{b}_j) / \bar{s}\,]$, we have:

$$\sum_{j=1}^{n} x_{ij} \ge \sum_{j=1}^{n} (a_i \underline{b}_j) / \underline{s} = \frac{a_i}{\underline{s}} \sum_{j=1}^{n} \underline{b}_j = a_i, \qquad (i = \overline{1, m}),$$

$$\sum_{j=1}^{n} x_{ij} \le \sum_{j=1}^{n} (a_i \bar{b}_j)/\bar{s} = \frac{a_i}{\bar{s}} \sum_{j=1}^{n} \bar{b}_j = a_i, \qquad (i = \overline{1, m}),$$

$$\sum_{i=1}^{m} x_{ij} \ge \sum_{i=1}^{m} (a_i \underline{b}_j)/\underline{s} = \frac{\underline{b}_j}{\underline{s}} \sum_{i=1}^{m} a_i = \underline{b}_j \frac{s}{\underline{s}} \ge \underline{b}_j, \quad (j = \overline{1, n}),$$

$$\sum_{i=1}^{m} x_{ij} \le \sum_{i=1}^{m} (a_i \bar{b}_j)/\bar{s} = \frac{\bar{b}_j}{\bar{s}} \sum_{i=1}^{m} a_i = \bar{b}_j \frac{s}{\bar{s}} \le \bar{b}_j, \quad (j = \overline{1, n}).$$

Thus, for each set of specified parameters from the given intervals, there exists a nonempty set of Q solutions corresponding to constraints (2) - (4). Moreover, according to the properties of (4) and (6), the polytop Q is a finite set of $m \times n$ Euclidean measures, which indicates the existence of a solution to the problem (1) - (6).

To construct the directional plan, the northwest corner method algorithm is applied, incorporating appropriate adjustments to account for parameter ranges. The development of the table used to generate the initial (preliminary) plan is carried out according to the following principles:

1. All requests (demands) must be satisfied.

2. Only the quantity of goods available in reserve may be dispatched to consumers.

If the interval representing a demand in the table has a defined left boundary, but there is no incoming supply flow, the left boundary value is assumed to be zero.

Now, based on the rules of 1^o, 2^o, we will describe the algorithm for determining the initial plan with the following example.

For example. Let it be $m = 2$, $n = 3$, $a_1 = 10$, $a_2 = 8$, $\mathbf{b}_1 = [2,5]$, $\mathbf{b}_2 = [3,8]$, $\mathbf{b}_3 = [4,5]$, $\mathbf{w}_{ij} \supseteq \mathbf{b}_{ij}$, $\forall i, j$.

1. Comparing $\mathbf{b}_1$ and a_1 gives $\mathbf{x}_{11} = [5,5]$, $\mathbf{x}_{13} = [0,0]$. Since it is $[10,\ 10] - [2,\ 5] = [5,\ 8]$, we assume that a_1 has a load left.

2. Comparison with the rest of the load at $\mathbf{b}_2$ a_1 $\mathbf{x}_{13} = [0,0]$, $[3,\ 8] - [5,\ 5] = [-2,\ 3]$ since the remaining request at $\mathbf{b}_2$ [0,3].

Thus, the basic plan of the problem looks as follows: $\mathbf{x} = \begin{pmatrix} [2,\ 5] & [5,\ 5] & 0 \\ 0 & [0,\ 3] & [4,\ 5] \end{pmatrix}$.

In the real case, the corresponding writing plan should be determined by many methods of a repetitive type, such as the simplex method, the potential method, etc. When considering intervals, the potential method turned out to be more convenient as the main real, as in this case the potentials can be differentiated themselves: the potentials from the suppliers' departure points can be real and the potentials (destinations) can be intermediate [8].

To describe the algorithm for solving the problem (1) - (6), we introduce some concepts.

Definition 2.3. Based on the location of the transportation problem in the interval settings, we refer to the following table 1.

Table 1.

	C_1	C_2	$\dots$	C_n	
A_1	$\frac{\mathbf{x}_{11}}{\mathbf{d}_{11}\mathbf{w}_{11}}$	$\frac{\mathbf{x}_{12}}{\mathbf{d}_{12}\mathbf{w}_{12}}$	$\dots$	$\frac{\mathbf{x}_{1n}}{\mathbf{d}_{1n}\mathbf{w}_{1n}}$	a_1
A_2	$\frac{\mathbf{x}_{21}}{\mathbf{d}_{21}\mathbf{w}_{21}}$	$\frac{\mathbf{x}_{22}}{\mathbf{d}_{22}\mathbf{w}_{22}}$	$\dots$	$\frac{\mathbf{x}_{2n}}{\mathbf{d}_{2n}\mathbf{w}_{2n}}$	a_2
$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$	$\vdots$
A_m	$\frac{\mathbf{x}_{m1}}{\mathbf{d}_{m1}\mathbf{w}_{m1}}$	$\frac{\mathbf{x}_{m2}}{\mathbf{d}_{m2}\mathbf{w}_{m2}}$	$\dots$	$\frac{\mathbf{x}_{mn}}{\mathbf{d}_{mn}\mathbf{w}_{mn}}$	a_m
	$\mathbf{b}_1$	$\mathbf{b}_2$	$\dots$	$\mathbf{b}_n$	

If, as a result of compiling the corresponding records plan using the northwest corner method, after comparing the request on C_j with the rest of the cargo on A_i, it is equal to one zero, then this element is called the base zero.

A cycle is a sequence of layouts with the following properties:

a) the first and last cells are the same;
b) both adjacent cells are located in the same row;
c) both adjacent cells are located in the same row;
g) There are no three cells in one row

$$\mathbf{w}_{ij} = [0, \max_{i,j}\{a_i, \bar{b}_j\}] \quad (i=\overline{1,\ m};\ \ j=\overline{1,\ n}) \ . \qquad (7)$$

Algorithm of interval potential method:

1. Based on the data from problem (1) - (6), we construct model $\mathbf{x}_{ij}=[0,\ 0]$ ($i=\overline{1,\ m}$, $j=\overline{1,\ n}$).

2. We form the reference plan $\mathbf{X}=(\mathbf{x}_{ij})$ using the northwest corner method based on the rules 1^0, 2^0, where the same cells of the layout are directed to $|\mathbf{x}_{ij}|\neq 0$ or the set of bases where the main zeros are located.

3. We calculate the potential u_i, $\mathbf{v}_j$ ($i=\overline{1,\ m}$; $j=\overline{1,n}$), so that the condition $|\mathbf{v}_j|\leq|\mathbf{d}_{ij}|$ ($i=\overline{1,m}$, $j=\overline{1,n}$) is met in any base cell of the model. To do this, we place $u_1=0$ and determine the consecutive remaining potentials.

4. We'll check the current plan for acceptability. If $u_i+|\mathbf{v}_j|\leq|\mathbf{d}_{ij}|$ ($i=\overline{1,\ m}$, $j=\overline{1,\ n}$) for all cells of the order, then if the plan is acceptable, move on to step 9.

5. To determine the beginning of the future traffic distribution cycle, we will look for a cell in the layout (p,q) will look like this

$$u_p+|\mathbf{v}_q|-|\mathbf{d}_{pq}|=\max_{i,j}|u_i+\mathbf{v}_j-\mathbf{d}_{ij}| \tag{8}$$

6. We delete all rows from the layout except for row p and column q, except for the main row. If there's a lot more to draw, repeat this process in the rest of the order.

7. From the interval $(p,\ q)$, we construct the cycle L. All cells except L must be primary.

8. Let the value of $\mathbf{x}_{ij}$ in the cell with the base zero be zero $t=\min_{i,j}\left\{\min\{|\underline{x}_{ij}|,|\overline{x}_{ij}|\}\right\}$, where the minimum is taken for all cells located in all locations of the cycle. We organize the "load shift over the recalculation cycle":

a) For all odd intervals, we multiply the transport by t as follows:

$$\mathbf{x}_{ij}:=\mathbf{x}_{ij}+[t,\ t], \tag{9}$$

b) The transport across all intervals decreases by t as follows:

$$\mathbf{x}_{ij}:=\mathbf{x}_{ij}-[t,\ t]\ . \tag{10}$$

In this case, all zeros obtained as a result of the recalculation according to (9) and (10) are primary except for the first. Thus, in the newly obtained plan, there will be $n+m-1$ basic intervals.

Next, we'll take step 3.

9. Write the optimal plan obtained as a result from the model in the form i, j, $\mathbf{x}_{ij}$ for all intervals $|\mathbf{x}_{ij}|>0$. For some uncertainty in the parameters, we calculate the cost of this plan, taken as an interval describing the maximum and minimum cost of transportation.

The algorithm of the potential method is structured and consists of the following parts: forming an initial plan using the "northwest corner" method; calculating potentials; verifying the feasibility of the current plan; identifying the cell - the starting point for traffic redistribution; eliminating rows for plan optimization; constructing a cycle; shifting the load to the recalculation cycle.

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