DESIGN AND DEVELOPMENT OF A MATHEMATICAL MODEL AND COMPUTATIONAL APPLICATION FOR CARDIOVASCULAR MODELLING

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Abstract. Mathematical modelling of the cardiovascular system is an important tool for understanding blood circulation, predicting disease progression, and improving clinical decision-making. The cardiovascular system consists of the heart, blood, and a complex network of blood vessels that transport oxygen, nutrients, hormones, and metabolic waste. Because direct measurement of every pressure, flow, and wall-stress variable in the human body is difficult, mathematical models provide a useful way to simulate cardiovascular function under normal and pathological conditions. This article reviews the main approaches used in cardiovascular and blood-vessel modelling, including zero-dimensional lumped-parameter models, one-dimensional pulse-wave models, three-dimensional computational fluid dynamics, and fluid–structure interaction models. It also discusses major equations, assumptions, applications, limitations, and future directions of cardiovascular modelling.

Keywords: cardiovascular system, blood vessels, mathematical modelling, hemodynamics, Windkessel model, computational fluid dynamics, fluid–structure interaction

Cardiovascular diseases remain a major global health problem, and mathematical modelling has become increasingly important for studying blood-flow behavior, vascular resistance, arterial stiffness, stenosis, aneurysms, hypertension, and heart-valve disease. Models allow researchers and clinicians to investigate how blood pressure, flow rate, vessel geometry, and wall elasticity interact during the cardiac cycle. The cardiovascular system is difficult to study because it is nonlinear, dynamic, and patient-specific. Blood flow changes continuously with heart contraction, vessel elasticity, vascular branching, and physiological regulation. Mathematical models simplify this complexity while preserving the most important physical and biological features. Reviews of cardiovascular modelling commonly classify models into 0D**,** 1D, and 3D approaches, depending on the level of spatial detail included. One-dimensional models describe blood flow along the length of vessels. They are more detailed than 0D models because they account for pressure waves, vessel area changes, and flow propagation. These models are especially useful for studying pulse-wave velocity,

arterial stiffness, and wave reflections in large arterial networks. Recent reviews emphasize that 1D models offer a useful balance between computational speed and physiological detail. Blood vessels are not rigid tubes; they deform under pressure. Fluid–structure interaction, or FSI, models combine blood-flow equations with vessel-wall mechanics. These models are important for studying aneurysms, arterial wall stress, valve dynamics, and vascular implants. The Navier–Stokes equations alone are insufficient when vessel-wall motion strongly affects blood flow. In FSI modelling, the fluid domain and solid wall domain are coupled. Blood pressure deforms the vessel wall, and vessel deformation changes the blood-flow domain. This feedback makes FSI models more realistic but also more computationally expensive. Most cardiovascular models rely on simplifying assumptions. Blood is often treated as an incompressible fluid because its density changes very little under physiological pressure. In large arteries, blood is frequently approximated as a Newtonian fluid, although in smaller vessels blood may show non-Newtonian behavior due to red blood cell interactions. Vessel walls may be assumed rigid, elastic, or viscoelastic depending on the model purpose.

Common assumptions include:

1. Blood is incompressible.
2. Flow is laminar in many normal arteries.
3. Vessel geometry may be idealized or patient-specific.
4. Boundary conditions represent inlet flow, outlet pressure, or vascular resistance.
5. Vessel walls may be rigid in simple CFD models or deformable in FSI models.

The accuracy of a model depends strongly on whether these assumptions match the physiological situation being studied. Mathematical modelling has many biomedical and clinical applications. First, models help explain normal cardiovascular physiology, including pressure-flow relationships, cardiac output, arterial compliance, and pulse-wave propagation. Second, models are useful in disease analysis. For example, stenosis can be studied by changing vessel diameter, hypertension can be represented by increasing vascular resistance, and aneurysm risk can be investigated through wall shear stress and wall-stress calculations. Third, patient-specific modelling is increasingly used to support diagnosis and treatment planning. Medical imaging such as CT, MRI, or ultrasound can be used to reconstruct vessel geometry, after which simulations can estimate blood flow, pressure drop, or intervention outcomes. Patient-specific cardiovascular mechanics has been reviewed as a way to predict the effects of alternative therapies for individual patients. Mathematical modelling offers several important advantages. It allows researchers to test hypotheses without invasive experiments, estimate variables that are difficult to measure directly, compare different treatment scenarios, and reduce the cost of experimental trials. It also supports

personalized medicine by adapting model parameters to individual patients. For example, CFD-based coronary models can estimate pressure and flow patterns in coronary arteries, while lumped-parameter models can simulate whole-body circulation quickly. Combining models at different scales can provide both systemic and local information. Despite its benefits, cardiovascular modelling has limitations. Model accuracy depends on reliable input data, including vessel geometry, blood viscosity, wall properties, heart rate, pressure, and flow boundary conditions. Many parameters are difficult to measure directly in patients. In addition, biological systems vary between individuals, and disease progression may involve biochemical, inflammatory, and cellular processes that are not always included in physical flow models. Another challenge is validation. A model must be compared with experimental or clinical measurements before it can be trusted. Three-dimensional and FSI models can be computationally expensive, while simpler 0D and 1D models may lack local detail. Therefore, the best model depends on the research question. Future cardiovascular modelling is moving toward multiscale, patient-specific, and data-driven approaches. Multiscale models combine 0D, 1D, and 3D methods to represent both whole-body circulation and local vessel behavior. Machine learning may help estimate parameters, accelerate simulations, and analyze large clinical datasets.

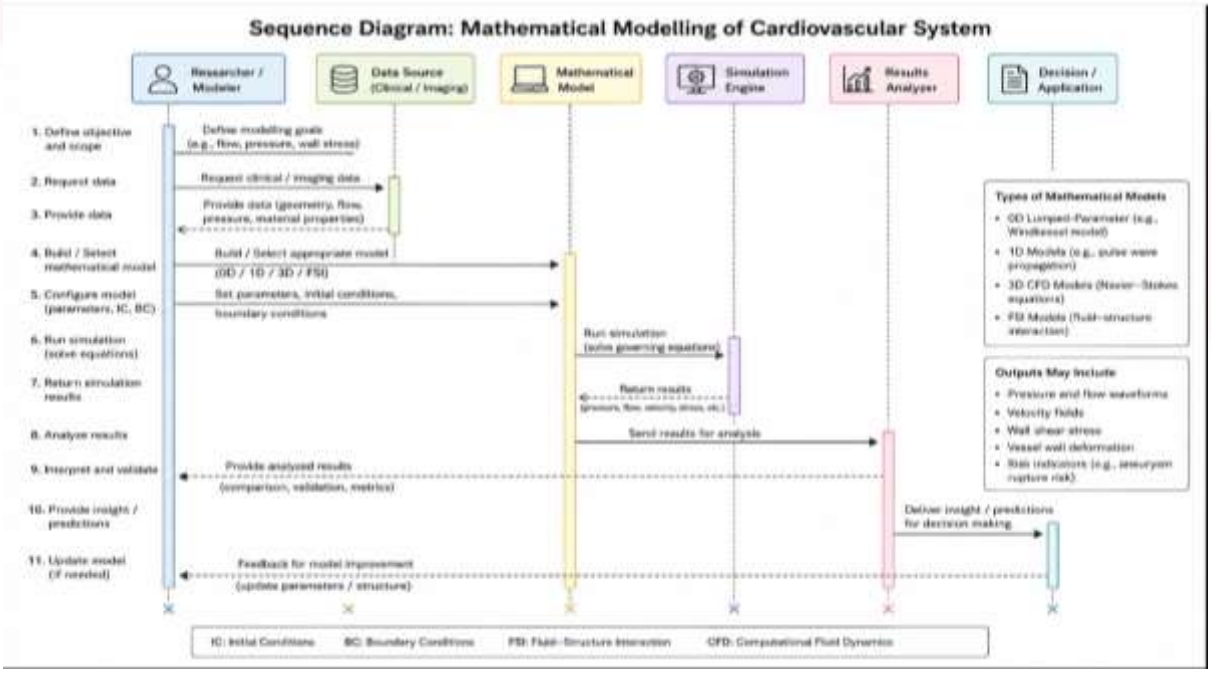

Fig1.1. Mathematical model of the cardiovascular system and blood vessels.

Then the model is configured by setting parameters, initial conditions, and boundary conditions. Initial conditions describe the starting state of the system, while boundary conditions describe values at the inlet and outlet, such as pressure or flow. The mathematical model is then sent to the simulation engine. The simulation engine solves the governing equations, such as blood-flow equations, pressure-flow equations, or Navier–Stokes equations. After the simulation runs, results are returned. These results may include pressure waves, flow waves, velocity fields, wall shear stress, vessel-wall

deformation, and risk indicators such as aneurysm rupture risk. The results analyzer checks and interprets the output. This may involve comparing the simulation results with clinical data, validating the model, and calculating important metrics.Finally, the analyzed results are delivered for decision-making. They can help doctors, engineers, or researchers understand cardiovascular behavior, predict disease progression, plan surgery, design stents or grafts, and improve treatment strategies. The last step is feedback. If the results are not accurate enough, the model can be updated by changing parameters, improving vessel geometry, or choosing a more advanced model. This makes mathematical modelling an iterative process. However, machine-learning models still require physiological interpretation and careful validation. Another promising direction is the development of digital twins of the cardiovascular system. A cardiovascular digital twin is a personalized computational model that can be updated with patient data and used to predict disease progression or treatment response. As imaging, wearable sensors, and computational power improve, mathematical modelling may become more integrated into clinical workflows. Mathematical modelling of the cardiovascular system and blood vessels provides a powerful framework for studying blood circulation, vascular disease, and treatment strategies. Zero-dimensional models are useful for whole-system pressure and flow analysis, one-dimensional models describe wave propagation in arterial networks, three-dimensional CFD models provide detailed local flow information, and FSI models account for vessel-wall deformation. Although challenges remain in parameter estimation, validation, and clinical translation, cardiovascular modelling continues to advance rapidly. Its future role is likely to expand in personalized medicine, surgical planning, medical-device development, and cardiovascular disease prediction.

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